Roll No. | | | | | | | | | | | | | **Total No. of Pages: 02**

Total No. of Questions: 09

B.Tech (Sem. – 1)

MATHEMATICS-I

Subject Code: BTAM-101-18

M Code: 75353

Date of Examination : 11-01-2023

Time: 3 Hrs. **Max. Marks: 60**

INSTRUCTIONS TO CANDIDATES:

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each, carrying EIGHT marks each.**
3. **Attempt any FIVE questions from SECTION B & C, selecting atleast TWO questions from each of these SECTIONS B & C.**

SECTION-A

1. Answer the following:

a) Give geometric interpretation of mean value theorem.

b) Can Rolle's theorem be applied to the function:

$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 \le x \le 2 \end{cases}$ in the interval $[0, \pi]$.

c) Evaluate $\lim_{x\to 0}[x^n(\ln\ x)]$.

d) Does the limit $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ exists?

e) Give the coordinates of the center of gravity of solid of mass *M*.

f) Define convergence, divergence and oscillation of a series.

g) Define D'Alembert's ratio test to check the convergence of the positive term series $\sum u_n$.

h) Find sum and product of Eigen values of the matrix $\begin{bmatrix} 1 & -1 \\ 2 & -5 \end{bmatrix}$.

i) Find the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$.

j) Find rank of the matrix $\begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix}$.

SECTION-B

2. a) Expand $f(x) = e^x$ in powers of $(x-1)$ upto four terms.

   b) Evaluate the limit $\lim_{x\to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x\sin x}$

3. a) Find the volume of the loop generated by the revolving the curve $y^2(a+x) = x^2(3a-x)$ about the x axis.

   b) Find extremum of the function $2\sin x + \cos 2x, 0 \le x \le 2\pi$.

4. a) Discuss the continuity of the function $f(x,y) = \begin{cases} \frac{x^2+y^2}{xy}, & (x,y) \ne (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ at (0,0).

   b) Find extreme values of $2x + 3y + z$ subject to the conditions $x + z = 1$ and $x^2 + y^2 = 5$.

5. a) Evaluate the integral $\iint_R e^{x^2}\, dxdy$, where R is the region given by $R: 2y \le x \le 2$ and $0 \le y \le 1$

   b) Evaluate $\iiint_T (x + 3y - 2z)\, dxdydz$, over the boundary of $T: 0 \le y \le x^2, 0 \le z \le x + y, 0 \le x \le 1$.

SECTION-C

6. Examine the convergence of the series $\sum \frac{3\cdot 6\cdot 9\cdots(3n)}{7\cdot 10\cdot 13\cdots(3n+4)} x^n$.

7. a) Examine the convergence of the series $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \cdots$.

   b) Examine the convergence of the alternating series $1 - \frac{1}{2^k} + \frac{1}{3^k} - \frac{1}{4^k} + \cdots$, for $k > 0$.

8. Find the characteristic equation of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and hence compute A^{-1}. Find the matrix represented by $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$.

9. Reduce the matrix $\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix}$ to the diagonal form.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.